Los 4 Amigos

Bridging Combinatorics and Bias Awareness in a Math Circle Activity

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MathFest 2025

NSF AISL 2215695

UCI Math CEO

- ❏ Math proficiency and motivation
- ❏ Socio-emotional learning
- ❏ Familiarity with college

Empowering Youth

Math Enrichment


HELLO
My Name is
Vans


6
Equity through Embracing Cultural Diversity
1
Students at the Center
Program Structure
Curriculum
Pedagogical Practices
Personal Development and Growth
5
Promoting Social Justice
2
Interpersonal Relationships and Well-being
4
Relevance, Friendly Language, and Representations
3
Deep Critical and Logical Thinking


6
Equity through Embracing Cultural Diversity
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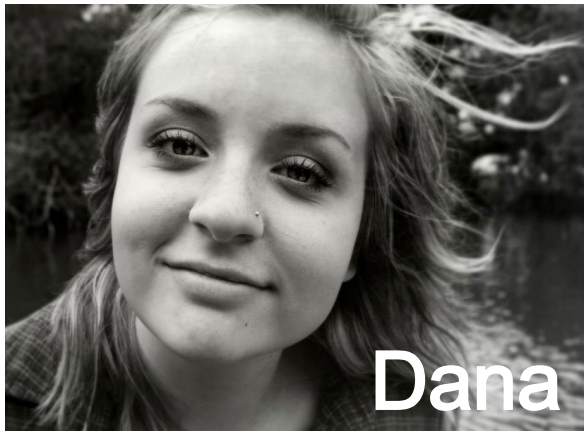
Interpersonal Relationships and Well-being
4
Relevance, Friendly Language, and Representations
3
Deep Critical and Logical Thinking

- Target: middle-schoolers
- Math Goals:
  - Develop counting strategies
  - Visualize counting (multiple representations)
  - Reasoning. Extend solutions to more complex problems
  - Make math collaborative
- Enrichment Goals:
  - Celebrate friendships through stories & experiences
  - Make biases visible
  - Enjoy challenges in a positive environment

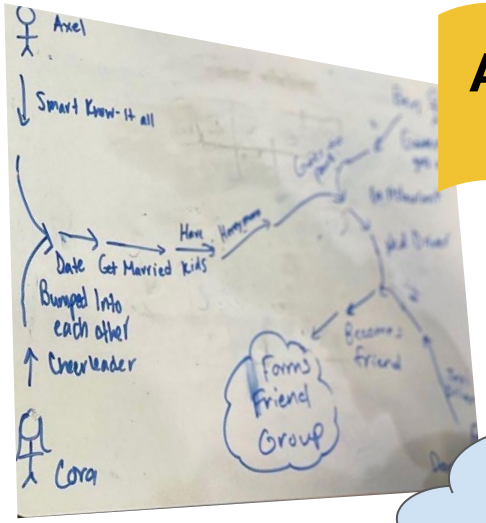
Los 4 Amigos

Create a world for these 4 friends:

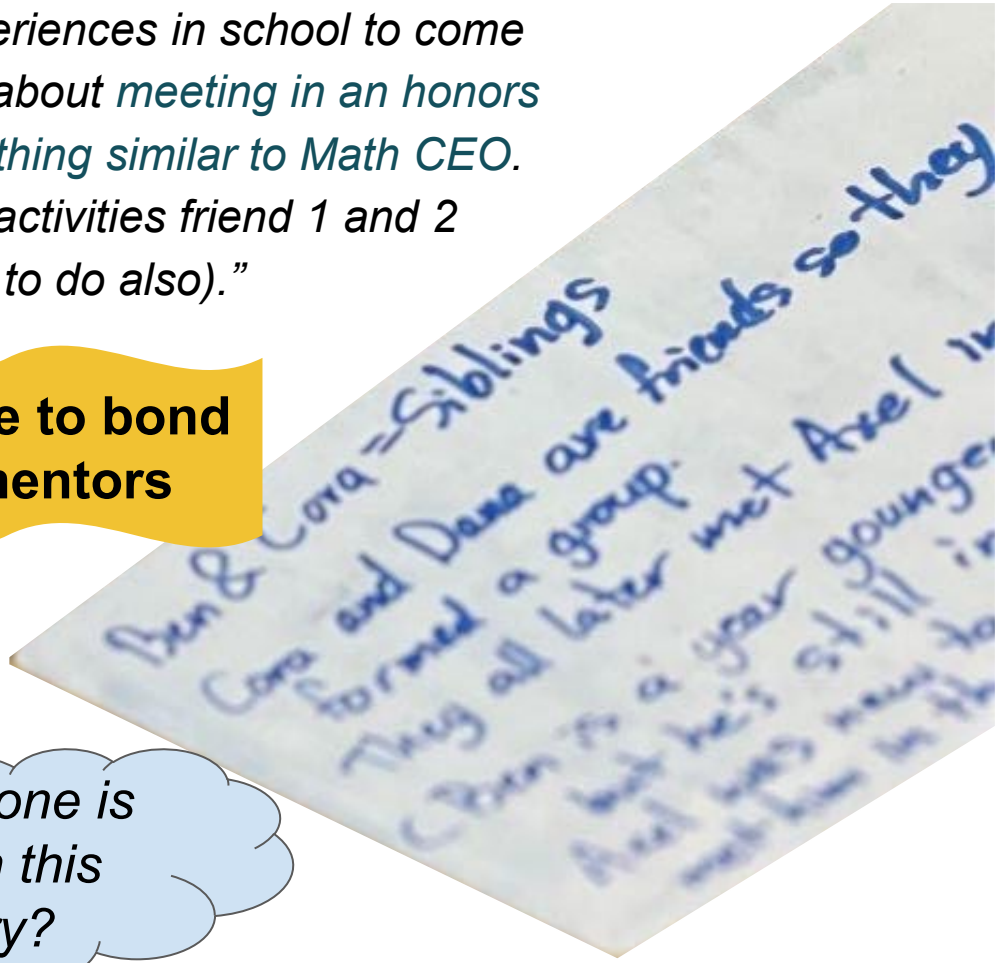
- *How do they know each other?*
- *What do they like to do together?*



"They all drew on their own experiences in school to come up with their story. Some talked about meeting in an honors history class or meeting at something similar to Math CEO. Another pair talked about which activities friend 1 and 2 could do (which is what they like to do also)."



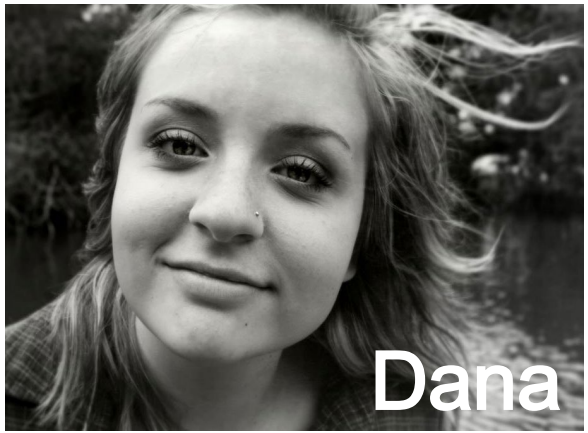
A chance to bond with mentors

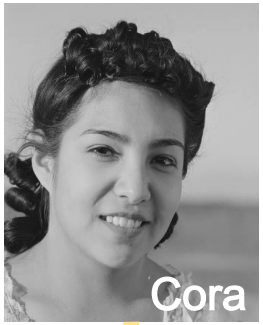


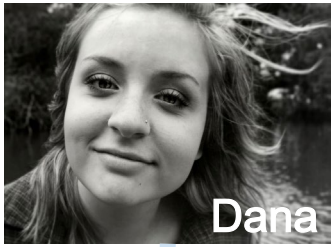
Which one is you in this story?

Match the 4 friends with the 4 hobbies

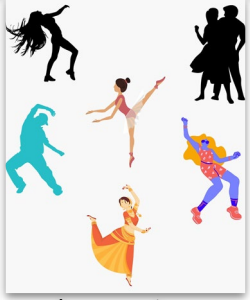
HOBBIES







video games



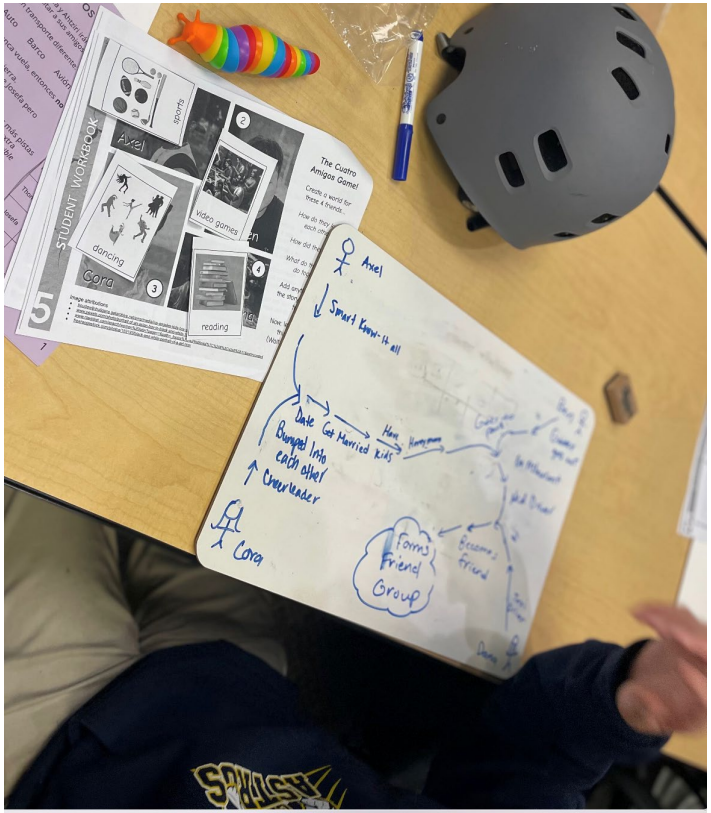
dancing

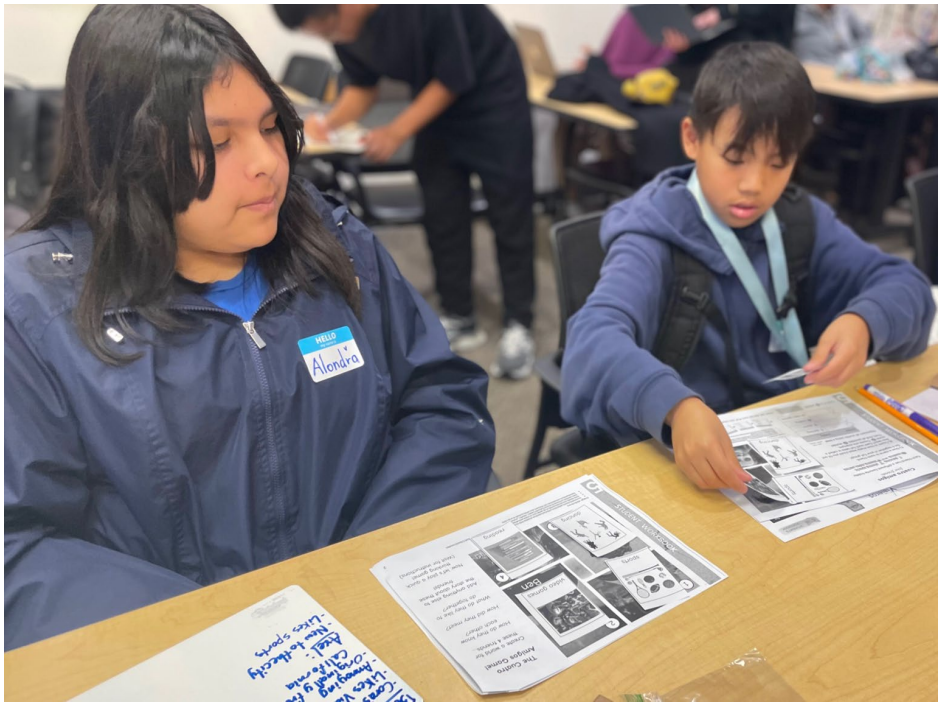
reading

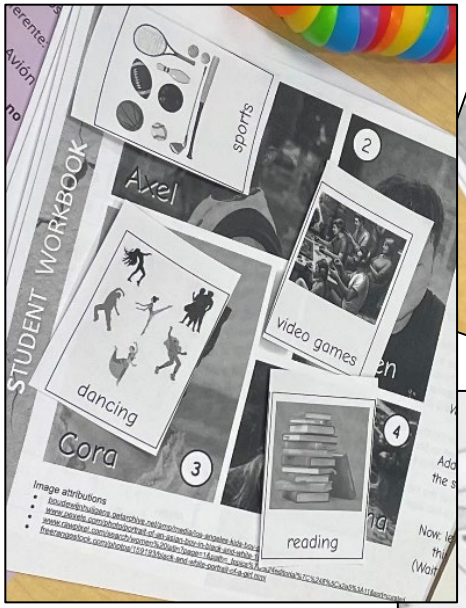
sports

Tally the results in the room

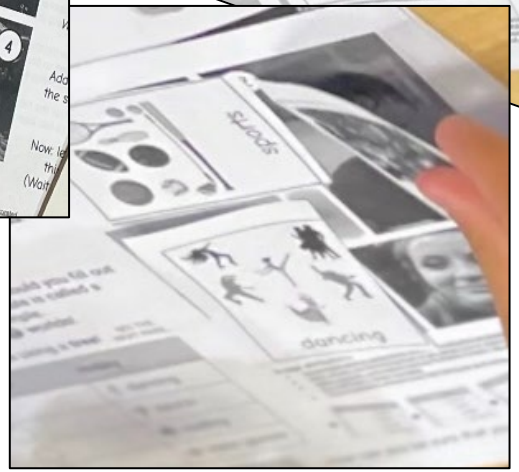
	dancing	sports	reading	video g.
1 				
2 				
3 				
4 				


STUDENT WORKBOOK
Axel
dancing
Cora
video games
reading
sports
The Cuatro Amigos Game!
Axel
Smart Know-it all
Date
Get Married
Have Kids
Bumped into each other
Cheerleader
Cora
Forms Friend Group
Becomes friend


HELLO
Alondra


STUDENT WORKBOOK
sports
Axel
video games
dancing
Cora
reading
Image attributions

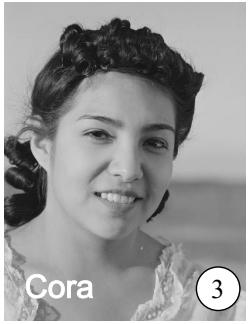

STUDENT WORKBOOK
sports
video games
Ben
dancing

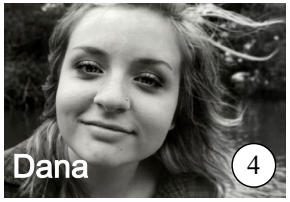

sports
dancing


dancing
sports
reading
video games


1
Axel


2
Ben


Cora
3

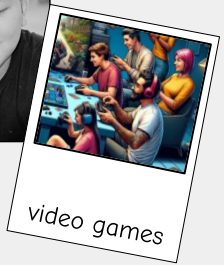

Dana
4

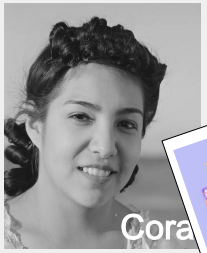

IMPLICIT
BIAS

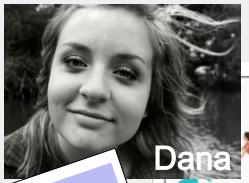

Axel


sports

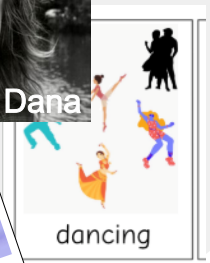

Ben


video games

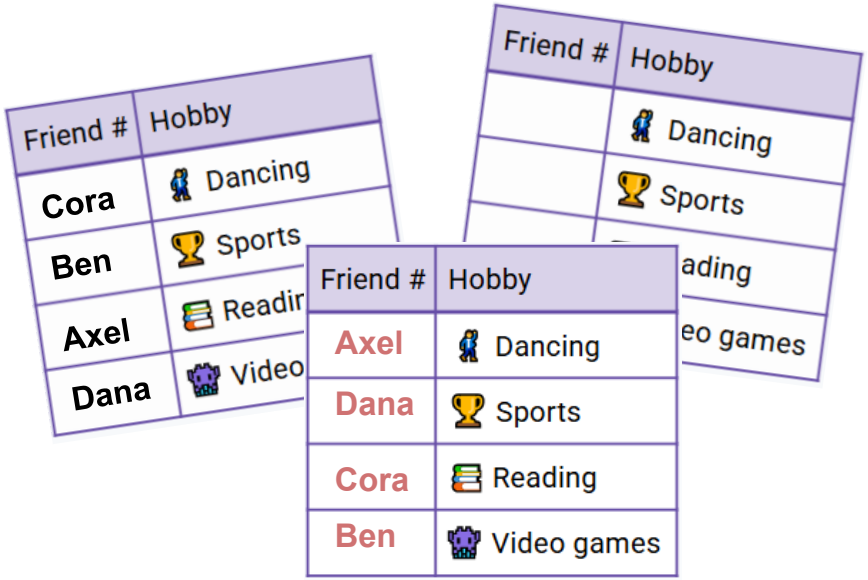

Cora


Dana


reading


dancing

Create all possible “worlds” for our 4 friends

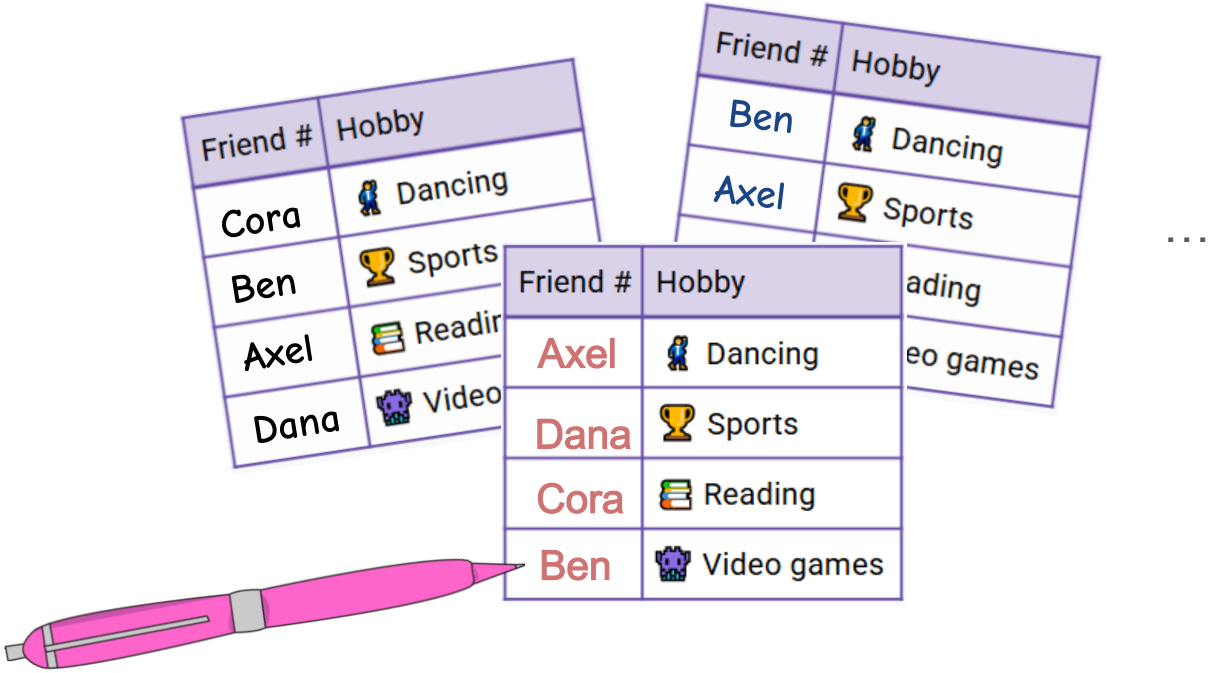


Each table represents “a world”
(a way to assign 4 hobbies to 4 friends with no repetition)

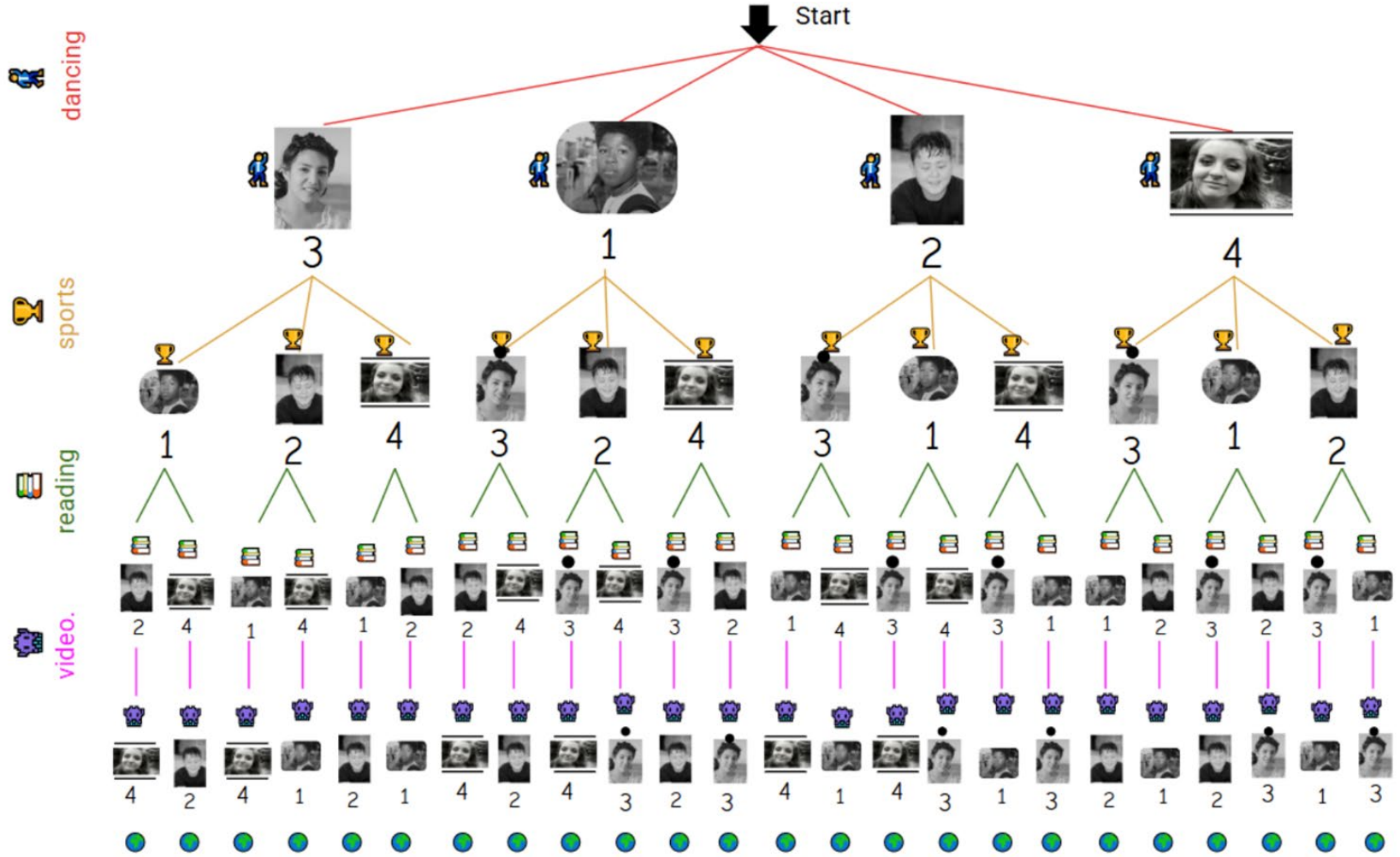
Set-up: Give 30 **blank** (tiny) tables to each student

Task: Fill in as many *distinct* cards as you can

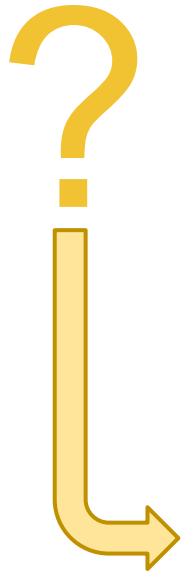
Expectation: No systematic approach yet

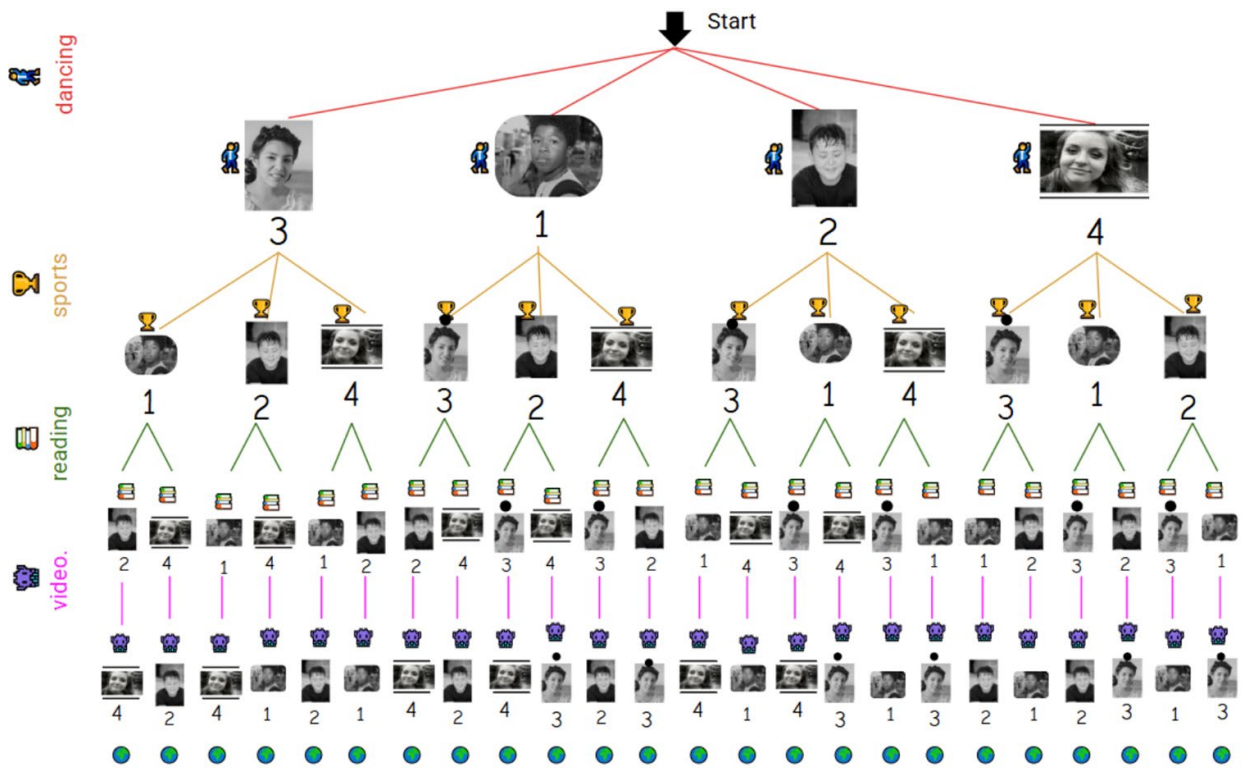


How do you know you found them all?


Start
dancing
sports
reading
video.
3
1
2
4
1 2 4 3 1 4 2 4 3 2 1 4 3 4 3 1 1 2 3 2 3 1
4 2 4 1 2 1 4 2 4 3 2 3 4 1 4 3 1 3 2 1 2 3 1 3

Friend #	Hobby
Cora	Dancing
Ben	Sports
Axel	Reading
Dana	Video games

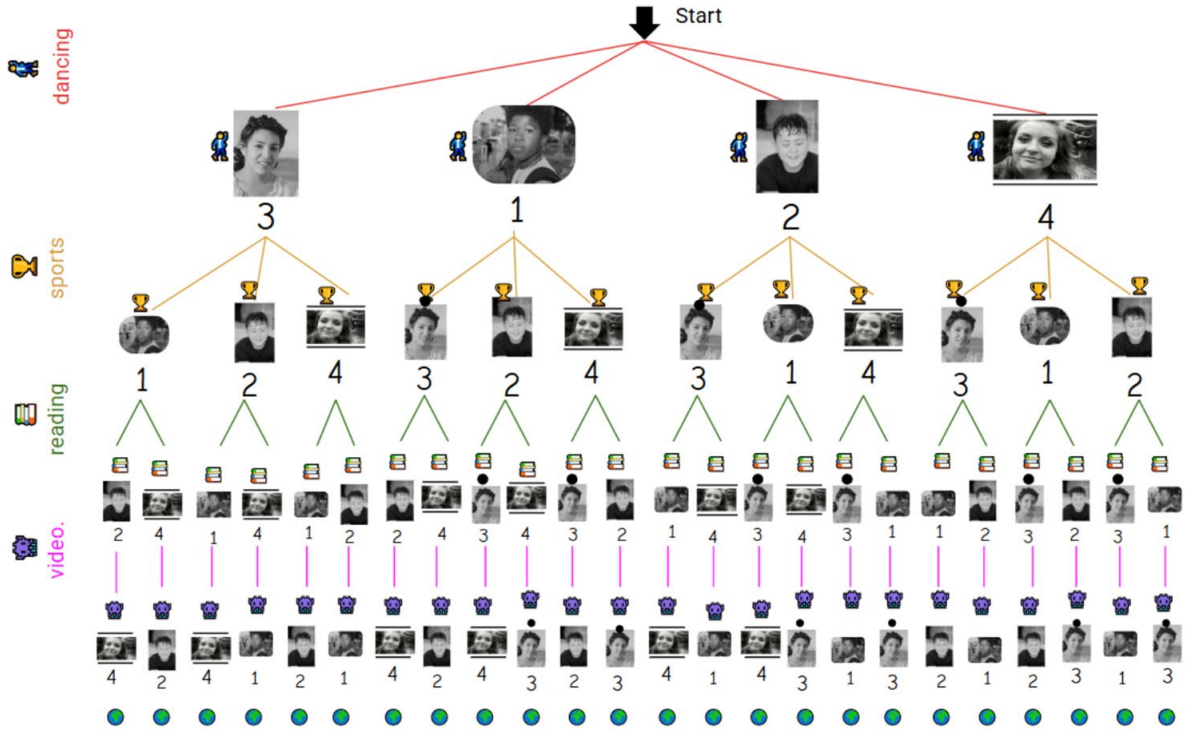




Which specific branch corresponds to this table?

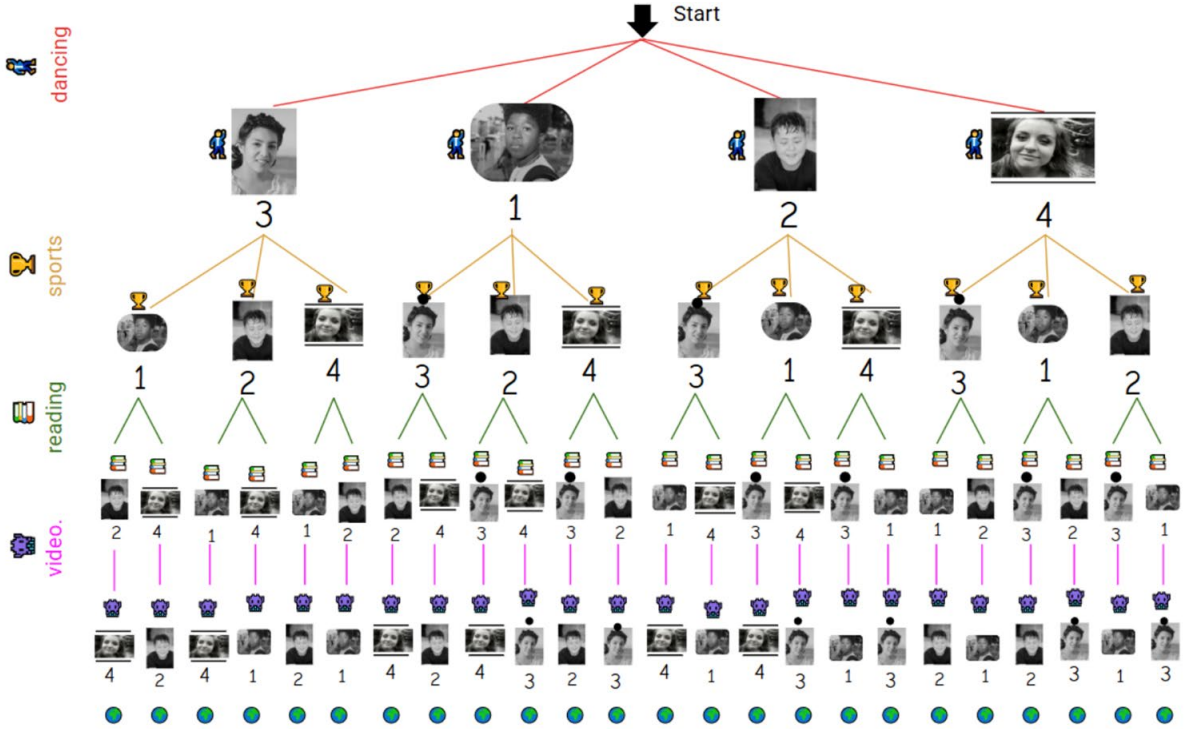
Are there more worlds in which Ben plays video-games, or worlds in which Ben reads?

What makes this world feel "unlikely"?

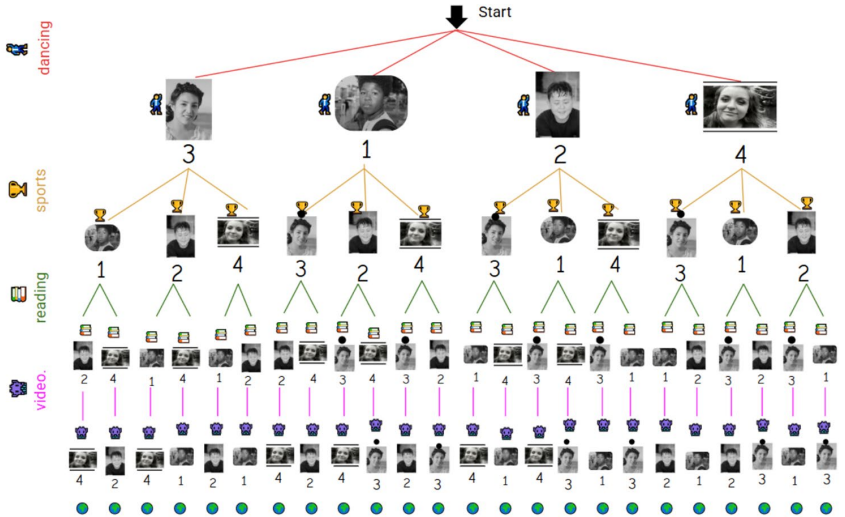


The ... worlds in total.

How many worlds are there in which Axel does NOT play sports?



Is this the only possible way to organize the tree?

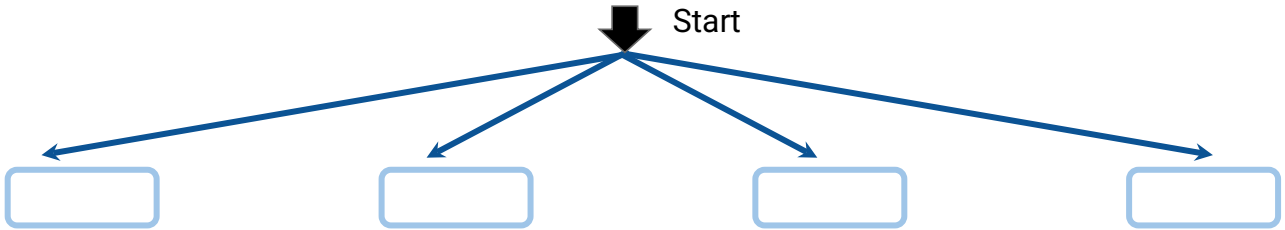


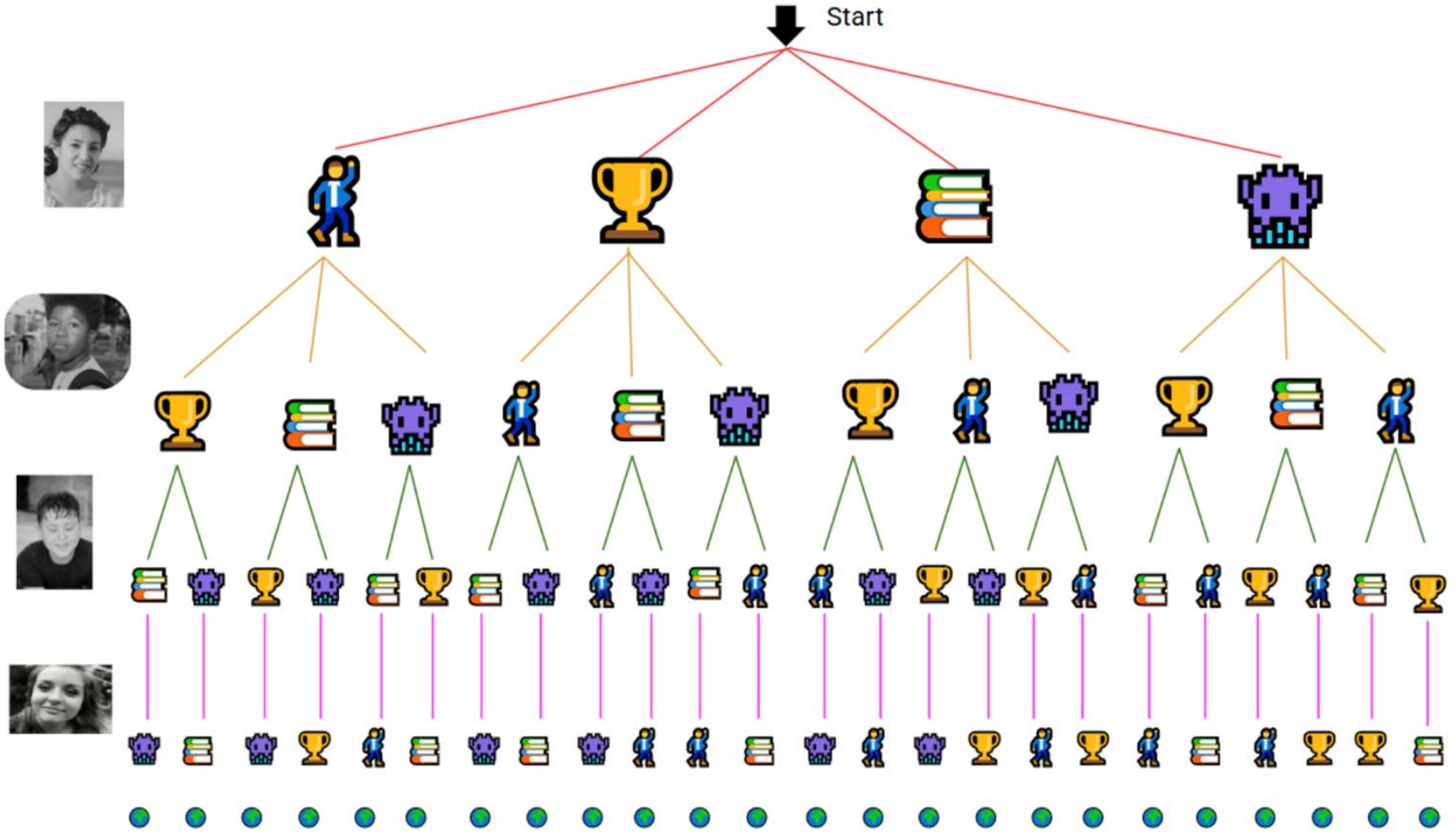
Flip the chart!

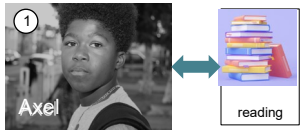
#1: For…

(4 options)

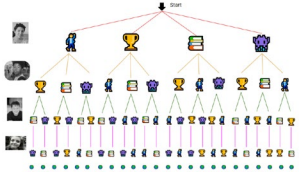
#2: For…




Start



Friend #	Hobby
	Dancing
	Sports
	Reading
	Video games



[4]→[3]→[2]→[1]

1st fr. 2nd fr. 3rd fr. 4th fr.

4 • 3 • 2 • 1 = 24

N•(N−1)•…•1 N!

Representation	Useful for…	Not that useful for…
Cards to match friend ←→ hobby	• Introducing the situation • Creating a world in a vivid, visual way	Making too many worlds (too time consuming)
Matching table (“A world”)	• “Seeing” a particular world, and listing many of them. • Comparing 2 worlds	• Checking that we built all worlds • Detecting combinatorial patterns
Counting trees (complete)	• Seeing all worlds at once • Seeing “subsets” fast • Detecting combinatorial patterns	Extending to more friends/hobbies (too time consuming!)
Counting trees (sketch)	• Extending to more friends/hobbies	When mastery is attained (can stop being helpful)
“Story-products”	• Communicating ideas efficiently • Making computations • Generalizing	Visualizing the situation at first (can be too abstract)
Expressions and equations		

Puzzle time! We are going to use some clues. Complete these 2 clues with 3 of the kids' names. *You can write each name at most once!*

1. _____ and _____ both think that dancing 🕺 is **not** amusing and sports 🏆 are **not** fun.
2. _____'s hobby is either playing video games 👾 or playing sports 🏆.

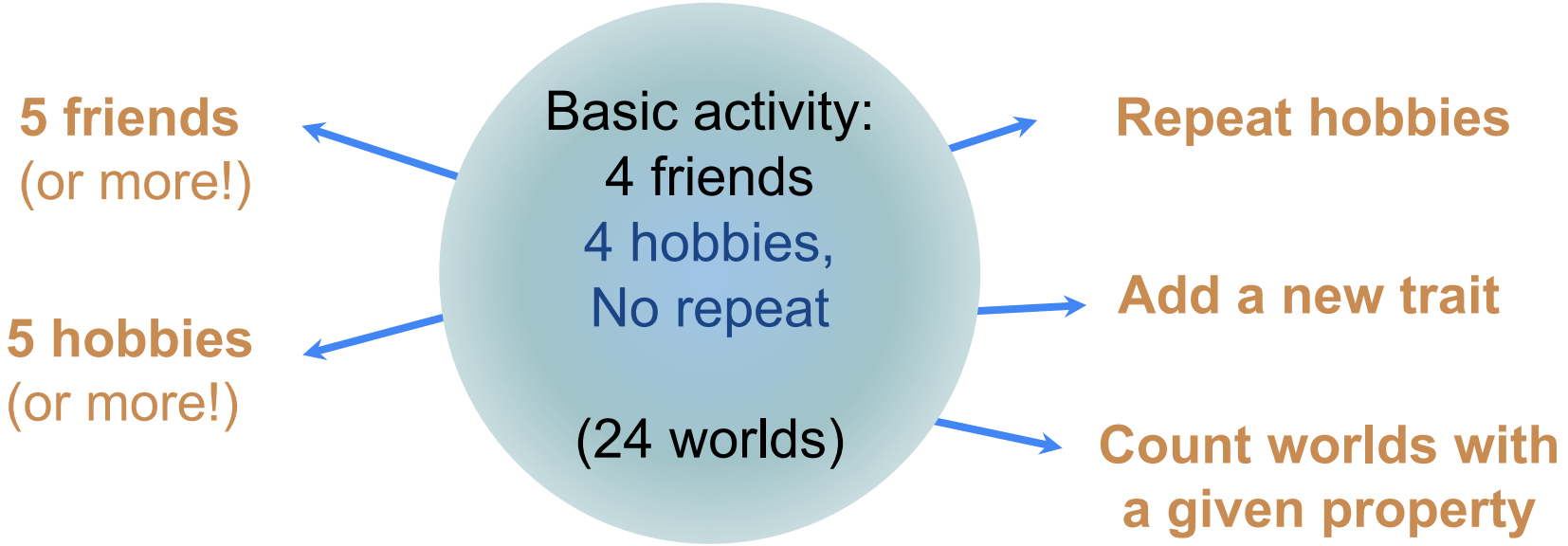
How many possible worlds satisfy both clues?

Write a third clue to help us narrow down the answer to just 1 🌍 solution!
(You can now repeat a person's name).

1. Final clue:

Friend	my deductions / possibilities	Hobby is...

Opportunities to Deepen Math Thinking



Increase the mathematical complexity

 # Friends: 4

 # Hobbies: 4

Repetition: N

 # Friends: 5

 # Hobbies: 5

Repetition: N

 # Friends: 10

 # Hobbies: 20

Repetition: N

 # Friends: 5

 # Hobbies: 4

Repetition: Y

 # Friends: 10

 # Hobbies: 3

Repetition: Y

Start with

- # Friends: 6
- # Hobbies: 10
- Repetition: Y

Then…

- .. Invite one more friend
- …or allow one more hobby

How does the answer increase?

!!

Reflecting About Today

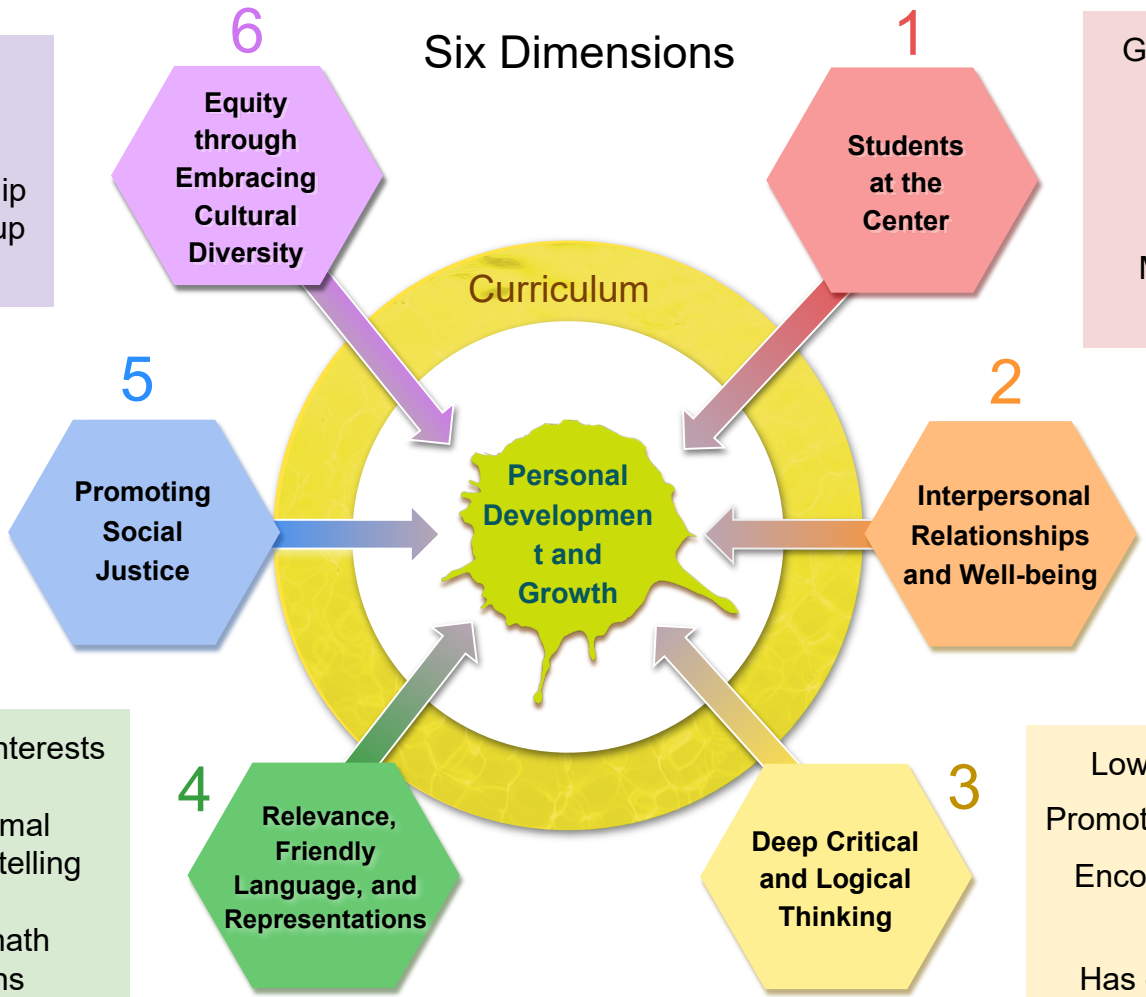
What did you learn about how different people think about **friendship**?

Why are trees **useful** when thinking about counting?

Why was the counting expression today 4x3x2x1 **and not 4+3+2+1**?

We all have **biases.** Were you surprised to find yours?

Six Dimensions



Helps surface stereotypes

Celebrates friendship among diverse group of students

Grounded in familiar topics (friendships, hobbies)

Collaborative, Joyful
Gives students agency

Manipulatives to support exploration

Raises awareness about bias

Makes all "worlds" equally possible

Allows for relationship-building (students share personal stories)

Includes "positive" hobbies

Reveals students' interests

Welcomes informal language & story-telling

Uses multiple math representations

Low-floor, high-ceiling tasks

Promotes combinatorial reasoning

Encourages relational thinking (not just procedural)

Has open-ended explorations

Thank you!

If you would like to use this activity, you can find it here…